

Roll Number		
-------------	--	--

Code Number 041/1/1



**INDIAN SCHOOL MUSCAT
THIRD PRELIMINARY EXAMINATION
MATHEMATICS**

CLASS: XII
01.02.2018

Sub. Code: 041
SET-1

Time Allotted: 3 Hrs
Max. Marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION – A (4 x 1 = 4)

Questions 1 to 4 carry 1 mark each.

1. Find x , if $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular. 1
2. Find the points where the function $f(x) = [x]$, $-2 \leq x < 3$ is not differentiable. 1
3. Find $\frac{dy}{dx}$, if $y = \cos^{-1}(\sin x)$ 1
4. Find the vector equation of a plane at which is at a distance of 7 units from the origin and the normal to the vector $(3\hat{i} + 5\hat{j} - 6\hat{k})$. 1

SECTION – B (8 x 2 = 16)

Questions 5 to 12 carry 2 marks each.

5. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, find $\frac{dy}{dx}$. 2
6. A balloon which is always remains spherical, has a variable radius. Find the rate at which its volume is increasing w.r.t its radius when the radius is 7cm. 2
7. Evaluate: $\int (\log x) dx$ 2
8. Find the points on the curve $y = x^3 - 11x + 5$ at which the tangent has the equation $y = x - 11$ 2

9. Find λ if the vectors $\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 9\hat{j} + 2\hat{k}$ are perpendicular. 2
10. Find the angle between the pair of the lines given by 2

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$
11. There are two types of fertilizers F_1 and F_2 . F_1 consists of 10% of nitrogen and 6% phosphoric acid and F_2 consists of 5% of nitrogen and 10% phosphoric acid. After testing the soil conditions a farmer finds that she needs at least 14 kg of nitrogen and 14kg of phosphoric acid for her crops, If F_1 costs Rs6/kg and F_2 costs Rs5/Kg. Formulate the problem so that nutrient requirements are met at a minimum cost. 2
12. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is exactly one defective egg. 2

SECTION – C (11 x 4 = 44)

Questions 13 to 23 carry 4 marks each

13. If $y = (x-2)^2(x+3)^2(x+5)^2$, find $\frac{dy}{dx}$. 4

OR
 Find the value of k, if $f(x) = \begin{cases} \frac{1-\cos kx}{x\sin x}, & \text{if } x \neq 0 \\ \frac{1}{2}, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.

14. Solve The Following: $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ 4

15. Integrate: $\int \frac{3x-2}{(x+1)^2(x+3)} dx$. 4

16. Evaluate the integral: $\int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$. 4

(OR)

Evaluate the integral: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$.

17. Prove that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$ 4

OR

Prove that $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

18. Solve :- $(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ given $y = 0$ when $x = 1$ 4
19. If with reference to the right handed system of mutually perpendicular unit vectors \hat{i}, \hat{j} and \hat{k} , $\vec{a} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to \vec{a} and $\vec{\beta}_2$ is perpendicular to \vec{a} . 4
20. $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, Find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} & $\vec{c} \cdot \vec{d} = 15$ 4
21. In an examination, an examinee either guesses or copies or knows the answer of multiple choice question with 4 choice. The probability that he makes a guess is $1/3$ and the probability he copies is $1/6$. Find the probability that his answer is correct, given that he copied it is $1/8$. Find the probability that he knew the answer to the question, given that he correctly answered it. 4
22. Two Cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces. Also find the mean. 4
23. Maximise $Z = 3x + 5y$, subject to the constraints: 4
- $x + 2y \leq 20$, $x + y \leq 15$, $y \leq 5$, $x, y \geq 0$.

SECTION – D (6 x 6 = 36)
Questions 24 to 29 carry 6 marks each

24. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1} \frac{1}{3}$. 6

OR

The length of the sides of an isosceles triangle are $9+x^2$, $9+x^2$ and $18-2x^2$ units. Calculate the value of x which makes the area maximum. Also find the maximum area of the triangle.

25. Let $f: \mathbf{N} \rightarrow \mathbf{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. show that 6
- $f: \mathbf{N} \rightarrow \mathbf{S}$, where \mathbf{S} is the range of f is invertible. Find the inverse of f .

OR

Let X be a non empty set. $P(X)$ be its power set. Let $*$ be an operation defined on elements of $P(X)$, by: $A * B = A \cap B$ for all $A, B \in P(X)$. Then

- Prove that $*$ is a binary operation on $P(X)$.
- Is $*$ commutative?
- Is $*$ associative?
- Find the identity element in $P(X)$ w.r.t. $*$
- Find all invertible element of $P(X)$.

f) If \blacksquare is another binary operation defined on $P(X)$ as $A \blacksquare B = A \cup B$, then verify that \blacksquare distributes itself over $*$.

26. Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$ 6
OR

Using integration, find the area of the following region: $\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$

27. Solve:- $\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y dx = \left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x dy$ 6

28. Find the equation of the plane through the intersection of the planes 6

$x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$.

OR

Find the length and the foot of the perpendicular from the point $(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$, also find image point.

29. 6

The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number, we get 12.

Using matrices, find numbers.

End of the Question Paper